Solutions, Work Sheet II

Anouncement:

we're planning a field trip to Fraunhofer ISE (Freiburg) on the 22nd of January, 2015. Please give me a short answer if you're interested to join it (michael.oldenburg@kit.edu). More details will follow.

1. **Density and thermodynamics of charge carriers**
   a) The integral is similar to the one obtained for photons in a black body:
   $$n_e(\epsilon_C) = \int D_e(\epsilon_e) f_e(\epsilon_e) d\epsilon_e$$
   The density of states can be calculated out of the total number of states:
   $$D_e(\epsilon_e) = \frac{\partial N_e}{\partial V \partial \epsilon_e}, N_e = 2\frac{4}{3}\pi\frac{p^3}{h^3}V$$
   The dispersion relation for electrons can be assumed to be the one for free electrons but with an effective mass $m^\star$:
   $$\epsilon_e = \epsilon_C + \frac{p^2}{2m_e^\star}$$
   Inserting this into the total number and differentiating with respect to the volume and the energy leads to the density of states:
   $$D_e(\epsilon_e) = 4\pi\left(\frac{2m_e^\star}{h^2}\right)^{3/2}\sqrt{\epsilon_e - \epsilon_C}$$
   The distribution of electrons is described by the Fermi-Dirac distribution function. Integrating over the energy leads to the following expression for the Density of charge carriers:
   $$n_e(\epsilon_C) = N_C \exp\left(-\frac{\epsilon_C - \epsilon_F}{k_B T}\right)$$
   The detailed calculation can be found in the book by Peter Würfel.

   b) Holes represent "missing" electrons and so:
   $$f_h(\epsilon_h) = 1 - f_e(\epsilon_e)$$

   c) To find the mean energy one needs to calculate the following integral:
   $$\langle\epsilon_e\rangle = \frac{1}{n_e}\int \epsilon_e D_e(\epsilon_e) f_e(\epsilon_e) d\epsilon_e = \epsilon_C + \frac{3}{2}k_B T$$
   This result is significant. It states that the electrons inside a semiconductor can be treated like an ideal gas. This is due to the assumption of the slightly modified dispersion relation. However, free particles once excited never relax to their previous state and so a recombination can only be modeled by additional time dependent terms.

2. Illuminated IV characteristic of a solar cell

For deriving the current of holes (electrons) one needs to start with the current density:

$$J_h = -qD_h \frac{dp}{dx}$$

An additional equation is the continuity equation:

$$q\frac{\partial p}{\partial t} + \frac{\partial j_h}{\partial x} = 0$$

Since the only local charge density change in an illuminated pn junction is due to charge generation or recombination processes it can be stated that:

$$\frac{1}{q}\frac{\partial j_h}{\partial x} = -(U - G)$$

Combining the first and last equation leads to the following differential equation:

$$D_h \frac{\partial^2 p}{\partial x^2} = U - G$$

The recombination rate U can be approximated by the relaxation time approximation (or more vivid, with an one-particle rate):

$$U = \frac{p_h - p_0}{\tau_h} = \frac{\Delta p}{\tau_h}$$

Here p_h is the excited hole density, p_0 the equilibrium hole density and τ_h the lifetime of an excited hole. *(This description of the recombination assumes that the number of excited states is low and there are enough states into which they can relax. However, for higher concentrations hole and electrons need to meet each other resulting in quadratic terms $U \sim p_h n_e$.)* Assuming that the equilibrium density distribution for holes is homogeneous in space the second order differential equation can be rewritten:

$$\frac{\partial^2 \Delta p}{\partial x^2} = \frac{\Delta p}{L_h^2} - \frac{G}{D_h}, L_h = \sqrt{D_h \tau_h}$$

In the lecture the generation rate was neglected. Yet, we are interested in the illuminated case. A general solution for Δp is:

$$\Delta p = G\tau_h + A \exp\left(\frac{x}{L_h}\right) + B \exp\left(-\frac{x}{L_h}\right)$$

Hera A and B are integration constants which can be derived from suitable boundary conditions like:

$$\lim_{x \to \infty} \Delta p(x) = const. \Rightarrow A = 0$$

$$\Delta p(0) = p_0 \exp\left(\frac{qV}{kT}\right)$$

The second condition was derived on the last work sheet (or in the lecture). Applying those constraints to the general solution gives:

$$\Delta p = G\tau_h + \underbrace{\left[p_0\left(\exp\left(\frac{qV}{kT}\right) - 1\right) - G\tau_h\right]}_{second\ constrain} \underbrace{\exp\left(-\frac{x}{L_h}\right)}_{first\ constrain}$$

Inserting this solution into the current expression gives:

$$j_h = qD_h \frac{\partial \Delta p}{\partial x} = q\frac{D_h p_0}{L_h}\left(\exp\left(\frac{qV}{kT}\right) - 1\right) \exp\left(-\frac{x}{L_h}\right) - qGL_h \exp\left(-\frac{x}{L_h}\right)$$

A similar expression can be found for electrons (with D_e, L_e). Until now we just included the generation rate over the whole region. However, to include the generation of holes inside the depletion region an additional term comes from the continuity equation:

$$\delta j_e = \delta j_h = q \int_{-W}^{0} (U - G) dx$$

It can be assumed that there is no recombination in the depletion region. So:

$$\delta j = qGW$$

The final solution for the current(!, not density) is:

$$I = I_0 \left(\exp\left(\frac{qV}{kT}\right) - 1 \right) - I_L, I_L = qAG(L_e + L_h + W)$$

3. **Solar Cell Design**

The equations we need for this task are:

$$P_{MP} = \eta P_{in}$$
$$\eta = \frac{V_{OC} I_{SC} FF}{P_{in}}$$

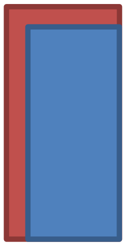
a) Here it is needed to find out P_{in} which is:

$$P_{in} = S \cdot A = 100 \frac{mW}{cm^2} \cdot A$$
$$A = 3 \cdot 13 \cdot (6\,cm \cdot 1{,}4\,cm) = 327{,}6\,cm^2$$
$$P_{in} = 32{,}76\,W \Rightarrow P_{MP} = 2W$$

b) Here the challenge was to calculate the surface correctly.

$$P_{MP} = \eta S A = 50\,W$$
$$A = 8333\,cm^2$$

This corresponds to 992 solar cells. Now we must include the area in between the solar cells:



For one cell this loss area is:

$$A_{singlecell} = 2{,}6\,cm^2 \Rightarrow A = 992 \cdot A_{singlecell} = 2530\,cm^2$$

So that the region of the whole module must be:

$$A = 1{,}1\,m^2$$

c) Here the second equation is needed. For a regular silicon solar cell the following assumptions can be made:

$$FF \sim 0{,}8, V_{OC} = 0{,}5\,V \,\left(J_{SC} = \frac{20mA}{cm^2}\right)$$

4. Anti-reflection coating

a) Using Fresnel's expression for the reflected amplitudes:

$$r_{01} = \frac{n_0 - n_1}{n_0 + n_1} = r_{12} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$n_1 = \sqrt{n_2 n_0}$$

This is the condition that the amplitudes of both light rays are the same. For air the refrective index is $n_0 = 1$ and for glass $n_2 = 1{,}46$. So the refrective index for the thin layer should be:

$$n_1 = \sqrt{1{,}46}$$

b) The phase difference between both rays must be $\Delta\phi = \pi$ or expressed in lengths:

$$\Delta x = k\frac{\lambda_0}{2}, k = 2N + 1, N \in \mathbb{N}$$

The ray passes twice the anti-reflexion coating and so:

$$2dn_1 = k\frac{\lambda_0}{2}$$